

AN ESTIMATION-BASED APPROACH TO MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) CHANNEL EQUALIZATION

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ABSTRACT

The purpose of this paper is to propose and investigate a new approach to implementing a spatio-temporal Decision Feedback Equalizer (DFE) for MIMO (Multiple-Input Multiple-Output) channels. A system with an array of n transmit and m receiver antennas where ($m \geq n$) is assumed. Both finite-length (Finite Horizon) and infinite-length (Infinite Horizon) MIMO Decision Feedback Equalizers are considered. We also assume an ISI (Inter-Symbol-Interference) MIMO channel, which means the channel matrix elements are frequency selective.

For the infinite-length case the DFE problem leads to solving a matrix spectral factorization. For the finite-length case the DFE problem leads to solving a corresponding Cholesky factorization.

Using the estimation-based spectral factorization we have shown that the solution to the infinite-length MIMO DFE is not unique. In the finite-length case the estimation-based approach leads to a recursive algorithm to perform the Cholesky factorization. The proposed recursive algorithm has low complexity and is also simple to implement. Moreover it leads to a closed form solution for the MIMO DFE matrices.

1. INTRODUCTION

Although the problem of channel equalization has been extensively studied in the literature the growth of wireless communications has presented new challenges. In particular, the introduction of space-time coding ([1],[2],[3],[4]) and application of antenna arrays at both transmitter and receiver ([5],[6]) has encouraged new research on equalization techniques for so called Multiple-Input Multiple-Output channels.

It must be noted that the problem of MIMO equalization has been readily studied in the literature ([7],[8],[9],[10],[11]). The purpose of this paper is to propose and investigate a new estimation-based approach to spatio-temporal equalization for MIMO (Multiple-Input Multiple-Output) channel equalization. We will consider a Decision Feedback Equalizers (DFE) structure. We have considered a system with n transmit and m ($m \geq n$) receiver antennas, therefore an $m \times n$ channel matrix. Each element of the resulting MIMO channel is considered to be frequency selective. The channel is also assumed to be AWGN (Additive White Gaussian Noise). The output of the m receiver antennas, after passing through the matched filter, are fed into the matrix equalizer(s). The DFE equalizer consists of two matrix of linear filters each with maximum L taps, one for the feedforward section and one for the feedback section. A perfect channel knowledge at the receiver is assumed.

For the infinite-length case the DFE problem leads to solving a matrix spectral factorization. For the finite-length case the DFE problem leads to solving a corresponding Cholesky factorization. We present estimation-based solutions to the vector spectral factorization and the corresponding Cholesky factorization, and these solutions are used to solve the corresponding formulated DFE problems.

We will show closed form solutions for the MIMO DFE matrices, which are due to the state-space structure used in solving the problem. None of the readily studied MIMO equalization methods lead to closed form solutions.

The paper is formatted as following. In section 2 we will discuss the MIMO DFE problem in two parts, the infinite horizon (infinite length) case and the finite horizon (finite length) case. In section 2.1 we will discuss the solution to the infinite length MIMO DFE. We will first present the matrix spectral factorization solution using a state space model in subsection 2.1.1. Then we will present the solution to the infinite length MIMO DFE problem in subsection 2.1.2. Correspondingly in section 2.2 we will discuss the solution

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to the finite length MIMO DFE. The matrix Cholesky factorization solution using a state space model is presented in subsection 2.2.1 and solution to the finite length MIMO DFE is presented in subsection 2.2.2. We finally conclude in Section 3.

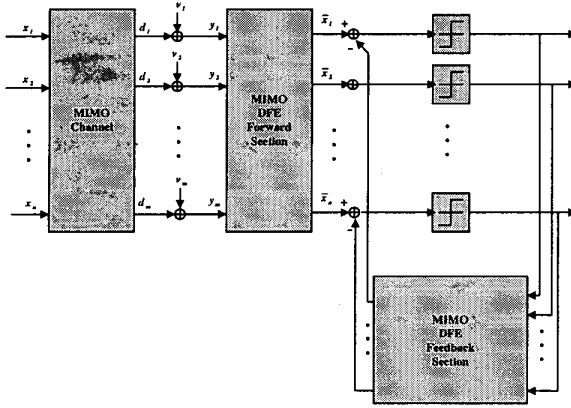


Figure 1: Block diagram of the MIMO DFE

2. MIMO DECISION FEEDBACK EQUALIZATION (MIMO DFE)

The structure of the MIMO DFE is shown in figure (1). Note that the variables in the figure are just used to depict inputs and outputs, and do not correspond to any specific variable in the paper. As in the scalar case the MIMO DFE equalizer consists of a feedforward, a feedback and a decision (slicer) section. Knowing the channel matrix the DFE problem is how to find the feedback and the feedforward matrices. In this paper the derivations will be based on the Minimum Mean Squared Error (MMSE) formulation. We shall note that MMSE formulation is stochastic versus the Least Square formulation which is deterministic. We will be considering two cases:

- The Infinite Horizon Case
- The Finite Horizon Case

2.1. Infinite Horizon

In the Infinite Horizon case the length of the DFE filters can be arbitrarily large, corresponding to the scalar infinite length DFE. The following derivations are discussed in [12], and we will just mention the final results here. Lets assume $\mathbf{Q}(z)$ represents the channel matrix convolved with

the matched filter matrix, and lets define $\tilde{\mathbf{Q}}(z)$:

$$\tilde{\mathbf{Q}}(z) = \mathbf{Q}(z) + \frac{N_0/2}{\epsilon_x} \mathbf{I} \quad (1)$$

where $N_0/2$ is the noise spectral density per dimension. Lets assume $\tilde{\mathbf{Q}}(z)$ has spectral factorization

$$\tilde{\mathbf{Q}}(z) = \gamma \mathbf{G}(z) \mathbf{G}^*(z^{-*}) \quad (2)$$

Then the MMSE feedback matrix $\mathbf{B}(z)$ is found as a result of the above spectral factorization to be

$$\mathbf{B}(z) = \mathbf{G}(z)^{-1} \quad (3)$$

The feedforward matrix $\mathbf{W}(z)$ is then found from $\mathbf{B}(z)$ (please refer to [12]).

2.1.1. Matrix Spectral Factorization

As discussed in the previous section in order to find the DFE matrices $\mathbf{B}(z)$ and $\mathbf{W}(z)$ we need to solve a matrix spectral factorization problem. In this section we will present a solution to the matrix spectral factorization using a state space approach used in [13]. We will present a matrix spectral factorization solution for a vector process whose statistics is described by a state space model. Then in the next section we will formulate our DFE problem, and will use the development in this section to solve the corresponding DFE spectral factorization problem.

Lets assume a process $\{y_i\}$ with a time-invariant space-time model

$$\begin{cases} x_{i+1} = F x_i + G u_i \\ y_i = H x_i + v_i \end{cases} \quad (4)$$

where $F \in C^{n \times n}$, $G \in C^{n \times m}$, and $H \in C^{p \times n}$ are known time-independent matrices, and $\{u_i\}$ and $\{v_i\}$ are zero-mean jointly stationary vector random variables that, along with the zero-mean random variable x_0 satisfy the conditions

$$\left\langle \begin{bmatrix} x_0 \\ u_i \\ v_i \end{bmatrix}, \begin{bmatrix} x_0 \\ u_j \\ v_j \\ 1 \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 & 0 \\ 0 & \begin{bmatrix} Q & S \\ S & R \end{bmatrix} d_{i,j} & 0 \end{bmatrix}. \quad (5)$$

As seen the driving processes $\{u_i\}$ and $\{v_i\}$ are white (temporally uncorrelated). For more information on the above state-space description please refer to [13].

When the process $\{y_i\}$ is stationary it can be shown that it has an exponentially decaying covariance function. We can therefore introduce its z-power spectral density matrix, $S_y\{z\}$, as the bilateral z-transform of $R_y(k)$,

$$S_y\{z\} = \sum_{k=-\infty}^{\infty} R_y(k) z^{-k}. \quad (6)$$

The state-space model (4) allows us to give some explicit formulas for $S_y\{z\}$. It can be shown that assuming $\{y_i\}$ being stationary then the z-spectrum is given by,

$$S_y\{z\} = K(z) \begin{bmatrix} GQG^* & GS \\ S^*G^* & R \end{bmatrix} K^*(z^{-*}) \quad (7)$$

where

$$K\{z\} = [H(zI - F)^{-1} \quad I] \quad (8)$$

It can be shown [14] that the z-spectrum $S_y\{z\}$ is not unique and there is an equivalence class for the input covariance matrices that give rise to the same z-spectrum, $S_y\{z\}$. In fact the non-uniqueness of the input covariance matrices was exploited to obtain the above spectral factorization of $S_y\{z\}$.

Given the above derivation we will summarize the matrix spectral factorization results in the following theorem referring from [14].

Theorem 1 (Matrix Spectral Factorization) Consider the time-invariant state-space model 4. Suppose that F is stable and that the initial state covariance matrix is such that

$$\Pi_0 = \bar{\Pi} = F\bar{\Pi}F^* + GQG^*, \quad (9)$$

so that the output process $\{y_i, i > 0\}$ is stationary. Assume further that $\{F, H\}$ is detectable, $\{F - GSR^{-1}H, G(Q - SR^{-1}S^*)^{1/2}\}$ is stabilizable, $R > 0$ and

$$\begin{bmatrix} Q & S \\ S^* & R \end{bmatrix} > 0.$$

Then the canonical spectral factorization of the z-spectral density function $S_y\{z\}$,

$$S_y\{z\} = K(z) \begin{bmatrix} GQG^* & GS \\ S^*G^* & R \end{bmatrix} K^*(z^{-*})$$

is given by

$$S_y\{z\} = L(z)R_eL^*(z^{-*}),$$

where $K(z)$ is given in (8) and the monic, causal and causally invertible transfer matrix $L(z)$ is given by

$$L(z) = H(zI - F)^{-1}K_P + I,$$

with

$$R_e = R + HPH^* \quad \text{and} \quad K_P = (FPH^* + GS)R_e^{-1},$$

where P is the unique positive semi-definite solution to the DARE

$$P = FPF^* + GQG^* - K_P R_e K_P^*.$$

Moreover, $F - K_P H$ is stable, which implies that $L^{-1}(z)$ exists and is stable.

2.1.2. Infinite Horizon DFE solution

As discussed in section 2.1 the solution for the feedback section is found from spectral factorization of $\tilde{Q}(z)$. In the previous section (2.1.1), we showed how to factorize $S_y\{z\}$ the z-spectrum of the stationary process $\{y_i\}$ which is described by the state-space model (4). In order to use the solution obtained in the previous section we will equate

$$S_y\{z\} \triangleq \tilde{Q}(z). \quad (10)$$

We have the solution to factorizing $S_y\{z\}$. Now we have to find the state-space model that leads to (10).

We will present the results and will leave the proof to [15]. It can be shown [15] that the following choices for the state-space model (4) will lead to (10):

$$F = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ I_m & 0 & \cdots & \vdots \\ 0 & I_m & 0 & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & I_m & 0 \end{bmatrix}, \quad (11)$$

$$H = [\tilde{Q}(1) \quad \tilde{Q}(2) \quad \cdots \quad \tilde{Q}(l)],$$

$$G = [I_m \quad 0 \cdots 0]^T$$

$$Q = 0, \quad R = \tilde{Q}(0) + \frac{I}{SNR}, \quad S = I.$$

where l is the length of $\tilde{Q}(z)$ in time. Therefore the DFE feedback matrix is $B(z) = L(z)^{-1}$ where $L(z)$ is found from Theorem 1 using the above state-space model parameters. The feedforward matrix $W(z)$ is then found from $B(z)$ (please refer to [12]).

2.2. Finite Horizon

In the finite Horizon case the length of the DFE filters are finite, corresponding to the scalar finite length DFE. The DFE structure is again as shown in Figure (1). As in the previous section we will leave the derivations to [12], and will just mention the results. As in the infinite length case the DFE solution depends on a matrix we denote by \hat{Q} and defined in [12] (\hat{Q} depends on channel matrix and noise spectral density). Lets assume \hat{Q} has Cholesky factorization

$$\hat{Q} = LDL^* \quad (12)$$

Then the feedback matrix B is found as a result of the above Cholesky factorization to be

$$B = L^{-1} \quad (13)$$

The feedforward matrix W then is found from B (please refer to [12]).

2.2.1. Matrix Cholesky Factorization

As discussed in the previous section the finite horizon (finite length) case, the MIMO DFE solution leads to a matrix Cholesky Factorization problem. We will use a similar state space approach used in the previous section to solve the matrix Cholesky factorization problem. Interestingly the state-space solution will lead to a Kalman filter solution to the matrix Cholesky factorization which in turn can be used to solve the Cholesky factorization encountered in the MIMO DFE problem.

Similar to the previous section we will present a Cholesky factorization solution for a vector process whose statistics is described by a state space model. Then in the next section we will formulate the DFE problem, and will use the development in this section to solve the corresponding DFE Cholesky factorization problem.

Lets assume a process $\{y_i\}$ with a time-varying space-time model

$$\begin{cases} x_{i+1} = F_i x_i + G_i u_i \\ y_i = H_i x_i + v_i \end{cases} \quad (14)$$

where $F_i \in C^{n \times n}$, $G_i \in C^{n \times m}$, and $H_i \in C^{p \times n}$ are known time-dependent matrices, and $\{u_i\}$ and $\{v_i\}$ are zero-mean jointly stationary vector random variables that, along with the zero-mean random variable x_0 satisfy the conditions

$$\left\langle \begin{bmatrix} x_0 \\ u_i \\ v_i \end{bmatrix}, \begin{bmatrix} x_0 \\ u_j \\ v_j \\ 1 \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 & 0 \\ 0 & \begin{bmatrix} Q_i & S_i \\ S_i & R_i \end{bmatrix} d_{i,j} & 0 \end{bmatrix}. \quad (15)$$

As seen the driving processes $\{u_i\}$ and $\{v_i\}$ are white (temporally uncorrelated). For more information on the above state-space description please refer to [14].

Lets assume we collect the process samples $\{y_i, i = 1, \dots, N\}$ in a vector y .

$$y = \text{col}\{y_0, y_1, \dots, y_N\}$$

and lets assume $R_y = \langle y, y \rangle$. The Cholesky factorization of R_y seeks a factorization of the form

$$R_y = LDL^* \quad (16)$$

where D is diagonal, and L is lower triangular. If we also impose the condition that L is lower triangular with unit diagonal, then under the additional assumption that all leading minors of R_y are non-zero, it turns out that such factorization always exists and are also unique. A white noise process can be obtained from the factorization

$$e = L^{-1}y, \quad R_e = L^{-1}R_yL^{-*} = D \quad (17)$$

where the vector $e = \text{col}\{e_0, e_1, \dots, e_N\}$ is also called the innovations process.

Now assuming that the $\{y_i\}$ process is modeled in state-space form (14), then the innovations process and therefore the Cholesky factorization of R_y can be found through the Kalman filter recursions [14]. Here we will refer to a theorem summarizing the final results of the Kalman filter recursions as developed in [14].

Theorem 2 (The Kalman filter recursions for the innovations)
Consider the state-space equations

$$\begin{cases} x_{i+1} = F_i x_i + G_i u_i \\ y_i = H_i x_i + v_i \end{cases}$$

where the $\{u_i, v_i, x_0\}$ are $m \times 1, p \times 1$, and $n \times 1$ -dimensional random variables such that

$$\left\langle \begin{bmatrix} x_0 \\ u_i \\ v_i \end{bmatrix}, \begin{bmatrix} x_0 \\ u_j \\ v_j \\ 1 \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 & 0 \\ 0 & \begin{bmatrix} Q_i & S_i \\ S_i & R_i \end{bmatrix} d_{i,j} & 0 \end{bmatrix}. \quad (18)$$

The matrices $\{F_i, G_i, H_i, \Pi_0, Q_i, S_i, R_i\}$ are assumed known. Then the lower triangular canonical factor L in the Cholesky factorization of R_y is given by:

$$L(i, j) = \begin{cases} 0 & i < j \\ H_i \Phi(i-1, j-1) K_{p,j-1}, & i > j \\ I, & i = j \end{cases} \quad (19)$$

where

$$\Phi(i, j) = \begin{cases} F_{i+1} \cdots F_j, & i > j \\ I, & i = j \end{cases}$$

$$\begin{aligned} K_{p,i} &= (F_i P_i H_i^* + G_i S_i) R_{e,i}^{-1} \\ R_{e,i} &= R_i + H_i P_i H_i^* \end{aligned}$$

$$P_i = F_i P_i F_i^* + G_i Q_i G_i^* - K_{p,i} R_{e,i} K_{p,i}^* \quad i > 0, \quad P_0 = \Pi_0$$

We note that, $P_i = \langle \tilde{x}_i, \tilde{x}_i \rangle$, where $\tilde{x}_i = x_i - \bar{x}_i$

2.2.2. Finite Horizon DFE solution

As discussed in the section 2.2 the solution for the feedback section is found from Cholesky factorization of \hat{Q} .

In the previous section (2.2.1), we showed how to factorize R_y the covariance matrix of the stationary process y which is described by a state-space model (14).

In order to use the solution obtained in the previous section we will equate

$$R_y \triangleq \hat{Q}. \quad (20)$$

We have the solution to factorizing R_y . Now we have to find the state-space model that leads to (20).

We will present the results and will leave the proof to [15]. It can be shown [15] that the following choices for the state-space model (14) will lead to (20):

$$F_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ I_m & 0 & \cdots & \vdots \\ 0 & I_m & 0 & \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & I_m & 0 \end{bmatrix}, \quad (21)$$

$$H_i = [\hat{Q}_2 \ \hat{Q}_3 \ \cdots \ \hat{Q}_l],$$

$$G_i = [I_m \ 0 \ \cdots \ 0]'$$

$$Q_i = 0, \ R_i = \hat{Q}_{11} + \frac{I}{SNR}, \ S_i = I.$$

where l is the length of \hat{Q} in time. Therefore the DFE feedback matrix is $B = L^{-1}$ where L is found from Theorem 2 using the above state-space model parameters. The feedforward matrix W is then found from B (please refer to [12]). The key observation used in the proof is that the matrix \hat{Q} has a block banded structure:

$$\hat{Q} = \begin{bmatrix} \hat{Q}_1 & \hat{Q}_2 & \cdots & \hat{Q}_l & 0 & \cdots & \cdots & \cdots & 0 \\ \hat{Q}_2 & \hat{Q}_1 & \hat{Q}_2 & \cdots & \hat{Q}_l & 0 & \cdots & \cdots & \\ \hat{Q}_l & \cdots & \hat{Q}_2 & \hat{Q}_1 & \hat{Q}_2 & \cdots & \hat{Q}_l & 0 & \\ 0 & \hat{Q}_l & \cdots & \hat{Q}_2 & \hat{Q}_1 & \hat{Q}_2 & \cdots & \hat{Q}_l & 0 \\ 0 & 0 & & & & & & & \\ 0 & \cdots & & \cdots & 0 & \hat{Q}_l & \cdots & \hat{Q}_2 & \hat{Q}_1 \end{bmatrix} \quad (22)$$

3. CONCLUSION

In this paper we proposed and investigated a new estimation-based approach to implementing a spatio-temporal Decision Feedback Equalizer (DFE) for MIMO (Multiple-Input Multiple-Output) channels. Both finite-length (Finite Horizon) and infinite-length (Infinite Horizon) MIMO Decision Feedback Equalizers were considered. For the infinite-length case the DFE problem lead to solving a matrix spectral factorization. For the finite-length case the DFE problem lead to solving a corresponding Cholesky factorization. Using the estimation-based spectral factorization we showed that the solution to the infinite-length MIMO DFE is not unique. In the finite-length case the estimation-based approach lead to a recursive algorithm to perform the Cholesky factorization. The proposed recursive algorithm is low complexity and is also simple to implement. Moreover it leads to a closed form solution for the MIMO DFE matrices.

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